Math 8, Summer 2012
Exam 1

Short Ans.	
1	
2	
3	
Total	

Name______________________________

Perm No.____________________________

Directions:

1. Each problem is graded out of 4 points.
2. Each short answer question is worth 1 point.
3. You're only allowed a writing instrument and your wits.
4. Proofs should be clean, to the point, and written in proper English sentences.

Short Answer

1. Given sets A and B, give a precise definition of $A \subseteq B$.

2. Let $f : A \to B$ be a function and $S \subseteq B$. Give a precise definition of $f^{-1}(S)$.

3. A sequence of continuous functions $f_1, f_2, f_3 \ldots$, each mapping from $[0, 1]$ into $\mathbb{R}$, is said to *converge uniformly* if and only if:

   For every $\epsilon > 0$ there exists $N \in \mathbb{N}$ so that for all integers $n, m \geq N$ and $x \in [0, 1]$ we have $|f_n(x) - f_m(x)| < \epsilon$.

   Give a precise statement of what it means for such a sequence not to converge uniformly.

4. Precisely define what it means for the function $f : A \to B$ to be surjective.

5. Given a collection of sets $\{A_i : i \in I\}$, precisely define $\bigcup_{i \in I} A_i$.

6. Which of these is not bijective?

   (a) The identity map $\mathbb{Z} \to \mathbb{Z}$
   (b) A 90° rotation of $\mathbb{R}^2$ about the origin
   (c) A translation of $\mathbb{R}^3$ by 3 units along an axis
   (d) The inclusion map $\mathbb{N} \hookrightarrow \mathbb{Z}$
   (e) None of the above

7. Given sets A and B, precisely define $A \times B$.

8. Give a precise definition of what it means for a real number x to be rational.

Problems

1. Let a and b be integers. Prove that $a + b$ is even if and only if $a^2 + b^2$ is even.
   Hint: There is a nice proof using $(a + b)^2$.

2. Let A and B be sets with $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$. Prove that either $A \subseteq B$ or $B \subseteq A$.

3. Suppose that $f : A \to B$ is an injective function and $S \subseteq A$. Prove that

$$f(A - S) = f(A) - f(S).$$